What's the Matter with the Deductive Nomological Model

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CSHPS annual conference
29 May, 2012, Waterloo
What’s inside

In this paper I:

1. Evaluate a possible extension of the Deductive-Nomological model to mathematical explanation(s)
2. Show that this extension lacks the resources to correctly account for mathematical explanations within its structure

Result of my analysis:

Highlight the reasons why such an extension, and more generally the picture of explanation on which the original D-N model is based, cannot handle mathematical explanations

... thus making the dissatisfaction towards the use D-N model in the context of mathematical explanation more ‘satisfactory’

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Two possible senses of Mathematical Explanation (Mancosu, 2008):

- **Mathematical explanation of mathematical facts** (MEM)
  → Explanation within mathematics (informal and formal proofs)

- **Mathematical explanation of scientific facts** (MES)
  → Explanations in the natural and social sciences where various mathematical facts play an essential role in the explanation provided.
In order for the proposed explanation to be sound, the constituents of the explanation have to satisfy certain conditions of adequacy (Hempel and Oppenheim, 1948, p. 137):

- **Logical conditions of adequacy:**
  - **R1** The explanandum must be a logical consequence of the explanans.
  - **R2** The explanans must contain general laws, and these must actually be required for the derivation of the explanandum and use no accidental generalizations.
  - **R3** The explanans must have empirical content: that is, it must capable, at least in principle, of test by experiment and observation.

- **Empirical condition of adequacy:**
  - **R4** The sentences constituting the explanans must be true.
Dissatisfaction towards the D-N model

In the context of scientific explanation:

- The D-N model does not provide sufficient conditions for successful scientific explanation (Bromberger 1966 and Salmon 1971)
- The D-N model does not provide necessary conditions for successful explanation (Scriven, 1962).

In the context of mathematical explanation:

Silence/Not clear! One reason for this ‘philosophical peacefulness’ may be that there is an extraordinarily obvious reason for it:

The D-N model mirrors the idea that the explanatoriness is not conveyed by the mathematics involved, thus ruling out the possibility of any genuine mathematical explanation (check condition of adequacy $R_3$: “The explanans must have empirical content, i.e. it must capable, at least in principle, of test by experiment and observation”).
I consider a possible extension of the deductive nomological model to the case of mathematical explanations in science. Alan Baker’s idea:

A broadening of the category of laws of nature to include mathematical theorems and principles, which share commonly cited features such as universality and necessity, would count the mathematical theorem as explanatory on the same grounds as the biological law (Baker, 2005, p. 235)
If we follow Baker’s suggestion, and we modify the conditions $R_n$ of the original D-N model, we obtain a possible extension (D-N Extended or D-N*):

$R_1^*$ The explanandum must be a logical consequence of the explanans

$R_2^*$ The explanans must contain general laws, which include mathematical theorems, and these must actually be required for the derivation of the explanandum.

$R_3^*$ The explanans must have empirical or mathematical content.

$R_4^*$ The explanans must be true

**Obs:** Hempel’s original idea that (sound) explanation is a matter of nomic expectability is preserved on this account.
D-N Extended: Dissatisfaction again

Problems:

$P_1$ it cannot deal with mathematical operations or procedures which play a key role in explanatory practices but which do not come under the form of statements (vs necessity)

$P_2$ it is not a sufficiently good indicator of the intuitions coming from the scientific practice, thus imposing a picture of explanation which is not authentic (vs sufficiency)
Example 1 (MES): Hénon-Heiles system(s)

[HH system]: particle moving in the Hénon-Heiles bidimensional potential

\[ U(q_x, q_y) = \frac{1}{2}(q_x^2 + q_y^2) + q_y q_x^2 - \frac{1}{3} q_y^3 \]  \hspace{1cm} (1)

Explanandum: behaviour of HH for low/high Energy.

There are two mathematical paths to study the system:

- Lagrangian analysis
- Hamiltonian formulation

Although both the routes are admissible, scientists agree that the mathematical procedure involving the Lagrangian formalism does not to convey the sense of explanatoriness that we obtain from the use of Hamiltonian formalism involving phase-space theory and a funcion called Poincaré Map (Lyon and Colyvan, 2008).
From this example it comes out that:

1. First, even if mathematics comes as essential ingredient, it is the particular mathematical structure involved which seems to convey explanatoriness (the Hamiltonian formalism including phase-space theory and Poincaré Map), and not a particular theorem (i.e. a mathematical law) \([P_1]\)
From this example it comes out that:

1. First, even if mathematics comes as essential ingredient, it is the particular mathematical structure involved which seems to convey explanatoriness (the Hamiltonian formalism including phase-space theory and Poincaré Map), and not a particular theorem (i.e. a mathematical law) \([P_1]\)

2. Second, although the two mathematical procedures are ‘deductively’ (and even ‘nomologically’) acceptable to indagate the physical phenomenon (regular or chaotic motion of the particle moving in the bidimensional potential), only one of them does contribute to the MES. Now, even if \(P_1\) is bypassed, according to the D-N Extended model both these procedures are equally explanatory \([P_2]\)
Example 2 (MEM):
Euler’s Theorem as it appears in Euler (1750).

To prove a geometrical theorem, Euler adds to the analytic proof a purely geometric proof.

According to Euler, the purely geometric proof is explanatory because, by making possible to reach the result through the same conceptual resources (geometrical concepts) which determine the content of the theorem (a geometrical concept), it provides the mathematician with the knowledge of the basic reasons why the result is true (see Molinini 2012 for the details).

⇝ However, again, according to the D-N Extended model both these proofs are equally explanatory [$P_2$]
The moral: the D-N framework (in his original and extended form)

- does not accurately describe important aspects of the explanatory practice carried out by scientists (vs necessity)
- does impose a criterion of explanation on the scientists, however scientists do not recognize this criterion as a ‘mark’ of genuine explanation (vs sufficiency)

**Diagnosis:** The failure of the D-N Extended model, as in the case of the original model, has do to with the inability of logic alone to mirror particular non-logical features of the mathematical explanations.

Making logical deduction the hallmark of explanation preserves the basic intuition of the D-N model but amounts to the imposition of a defining characteristic feature on what ought to be counted as “mathematical explanation”. The resulting picture of explanation is incomplete and not satisfactory.
Acknowledgements

... THINK IN PROGRESS

Thank you!

Presentation [with Bibliography] available here:


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