String figures
Recent outcomes

1. Introduction (briefly)

a. "String figures": an ethnomathematical object

- Definition : films + pictures

b. 2 conceptualizations (see Eric's paper for RHM)

- Elementary operations and sub-procedures
  - . Sub-procedures and iteration
  - . Sub-procedures and patterns
- Normal positions (a few words)

  Initial position-opening- normal positions + passages - final figure
- Mathematical ideas (algorithms, geometrical investigations in complex spatial configurations, itaration, transformation, …..)

2. String figures on the field

a. String figures and Anthropology (Céline)
b. Céline's field work(Iglulik, Nunavut)
c. Eric's field works (Marquesas, Polynesia – Chaco, Paraguay)
d. How can Céline and Eric collaborate ?

3. More recently

a. Thomas Storer's mathematical work on string figures
   - String figures' calculus
   - "Heart-sequence" of a string figure

a. Exploration of the Solomon string figure *Niu*
b. The concept of "prefix"
c. The system of Solomon string figures Patterns
d. Understanding of the transformation *Au kape* to *Pu kava.*

a. Definition

To play, you first knot the ends of a one to two meter long string making a loop: the game then consists of a succession of manipulations using the fingers and the string, but also at times including the teeth or feet. You finish with a final figure which you show to others around you.

Example: Kety, from Laguna negra, Chaco, Paraguay, making the final figure *Avestrus*

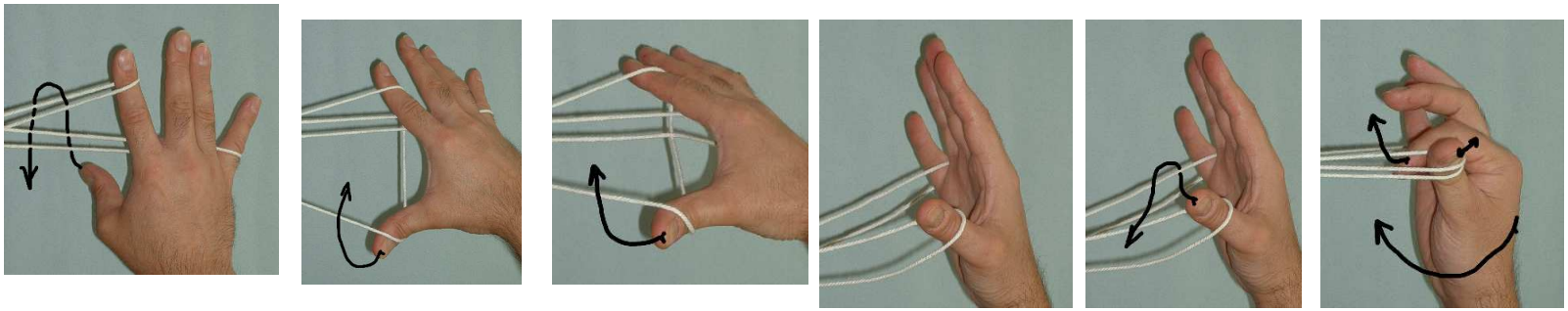
b. 2 conceptualizations
(see Eric's paper for RHM)

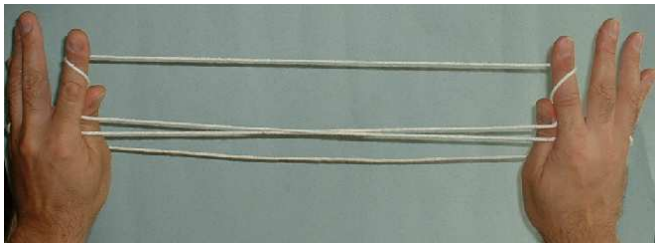
- Elementary operations and sub-procedures

- A string game can always be described by a succession of simple gestures or elementary operations. You can note the movement of taking up, hooking (these two operations result in the formation of a loop (open in the first case, closed in the other), extending a string, dropping a loop or creating a finger loop. In this way, one can say that any figure obtained is the result of a certain number of **elementary operations**.
- Example : *Niu*
- In addition to these elementary operations, one can identify **sub-procedures**, that is to say a succession of elementary moves shared in the same way by several string figures.
- Examples : openings, transfer, caroline extension (ending sub-procedure or not : see *Ashes*)…

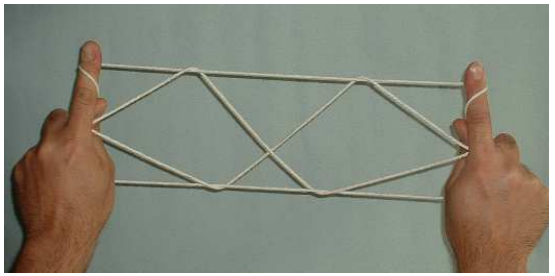
- Sub-procedure and iteration

PNG string figure *family sickness*





SP 1 →



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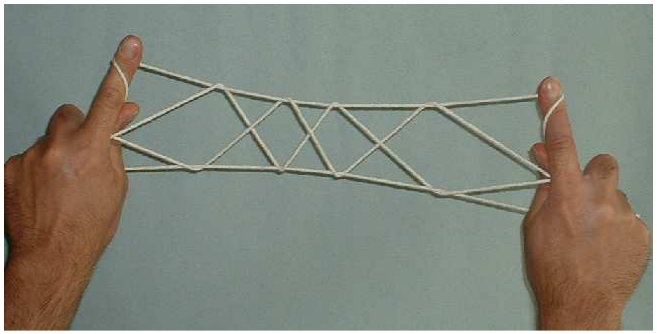

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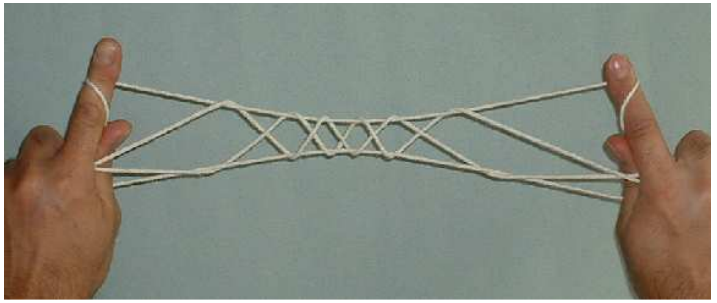
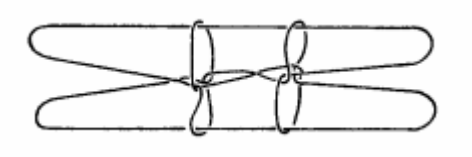


-String figures and patterns

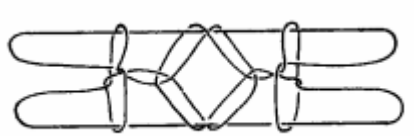
- Example : when looking at certain string figures collected in the 20th century in the Solomon Islands, you are struck by the fact that the final forms of the figures seem to have been conceived from a few simple patterns, and that the same “pattern” can be used many times over in other designs.

Some patterns from Solomon Islands

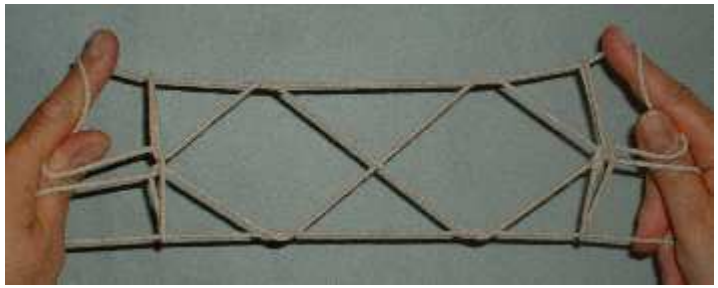
Roke nioke ke



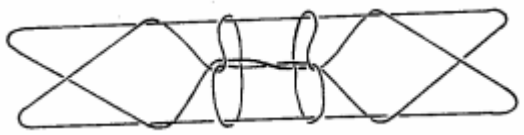
Namu



Repi susuhe'u



Whai wane



- Normal positions

J. Braunstein writes: “We will speak about ***normal position*** **when the hands hold a continuous string taut** ” and he calls “passage” any succession of operations which modifies the position of the string between two returns back to “normal position”.

The definitions of these various states enable him **to conceive a string figure like a “sequence”**, which begins with an *initial position*, followed by an *opening* which puts the string in *normal position*. Then, a series of *passages* is then applied between some *normal positions*, and finally the *final figure* concludes the message.

Example: *Pilun* from PNG

- Mathematical ideas

. Algorithms

The elaboration of procedures results in the intellectual process of organizing elementary operations. They therefore constitute algorithms. Working in this way consists in fact of identifying ordered sets of operations - the sub-procedures - having a significant impact on different substrata (i.e. particular positions of the strings)

. Geometrical investigations in complex spatial configurations

- Modification of one and only one operation
- Two procedures, one longer than the other, for the same final figure

. Iteration

Ex : See *Family sickness*

. Transformation

Ex : PNG string figure *Umaha*

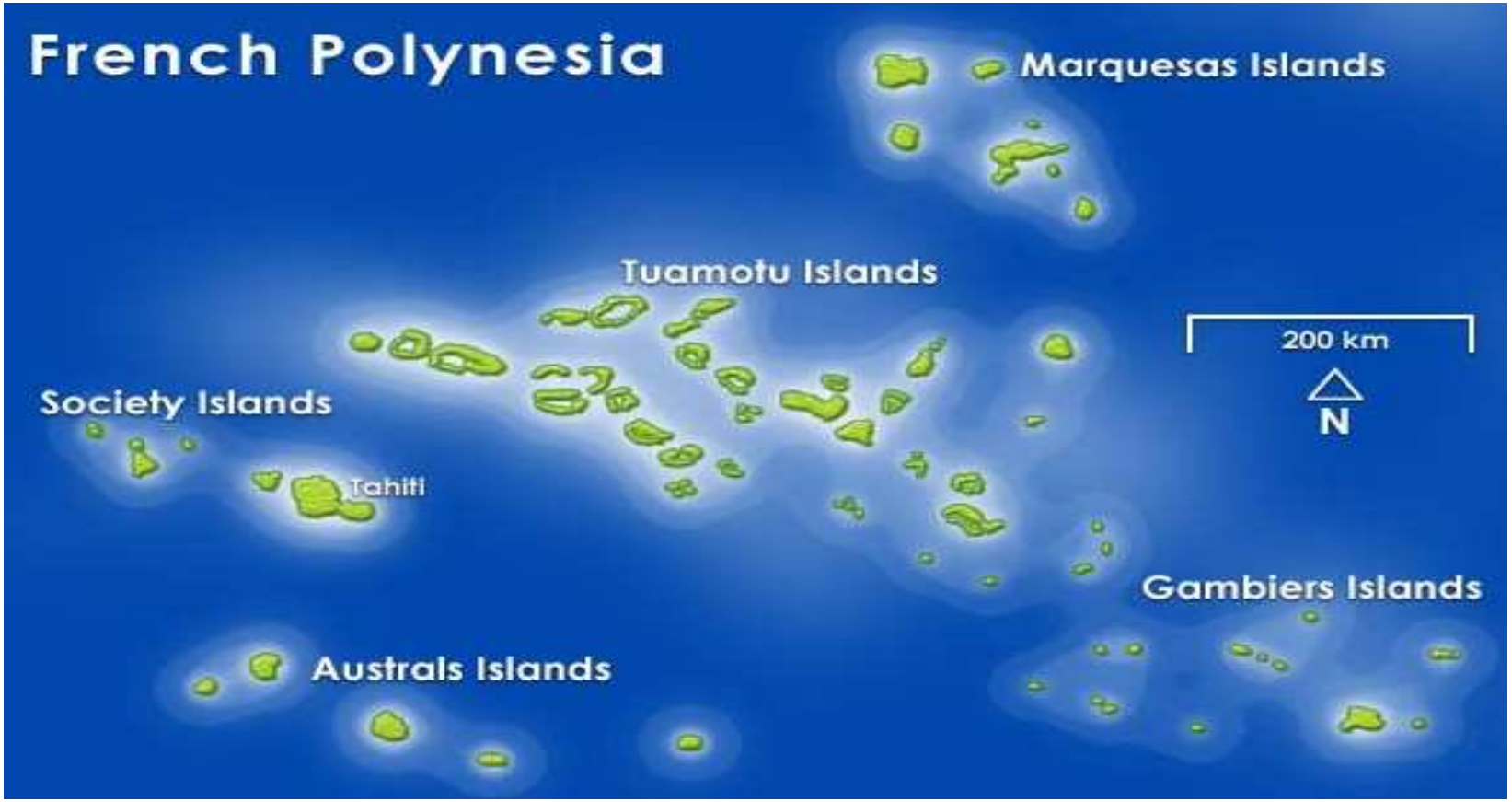
2. String figures on the field

a. String figures and anthropology

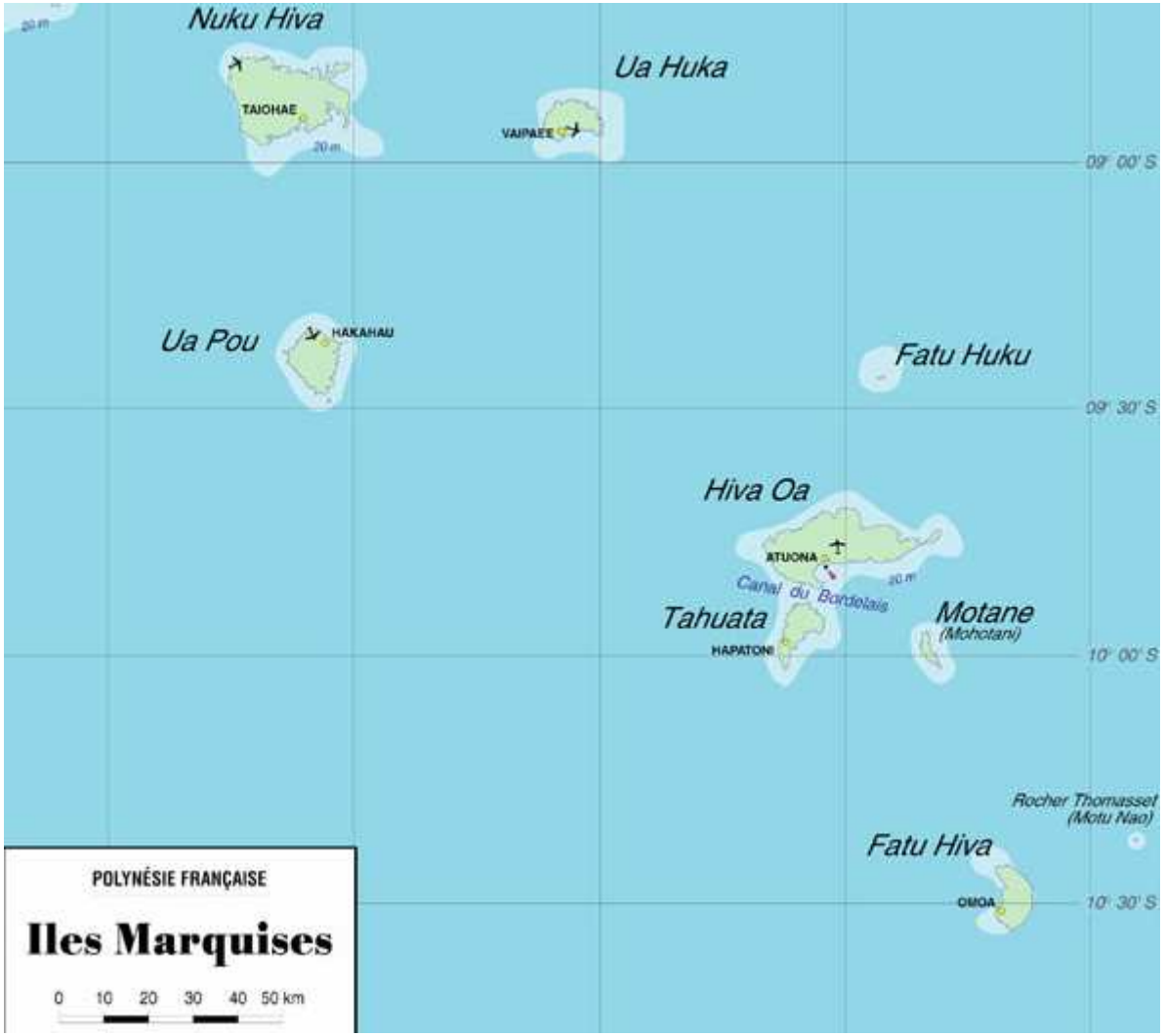
Céline

b. Céline’s field work

c. Eric's field works



Marquesas Islands



Ua Pou

Map of *Ua Pou*


RT 455
RT 456


Hakaetau
Hakahau
Haakuti
Hakamui
Hakamaii
Hohoi
Hakatao

The practices of *Pehe* (string figures) on the island of *Ua Pou*.

- Knowledge still held by the elders
- A widely practised activity about fifty years ago by individuals of all ages
- Reasons of their disappearance
- My investigations (informants, reception, first results,….)

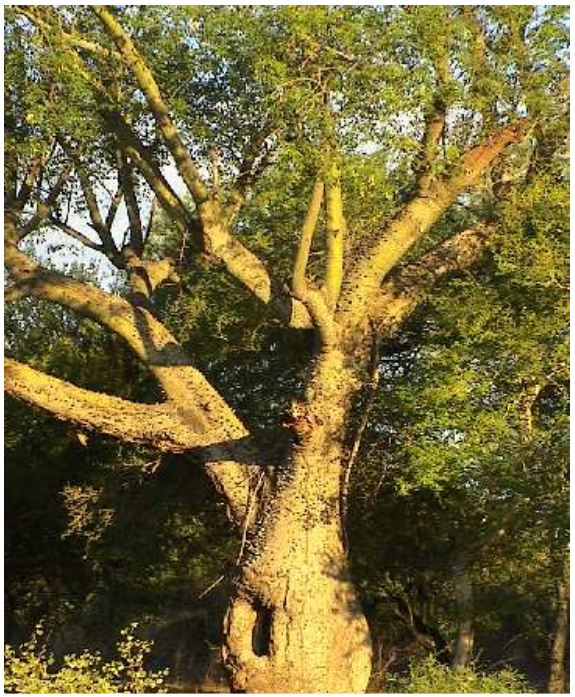
Santa Teresita, Chaco, Paraguay

The practices of *Juegos de hilo* in the Chaco

- My investigations were carried out in *Santa Terisita* and also in the village *Laguna negra* located 30 kms away, both inhabited by Guarani-Nandeva Indians
- The practice of making "string figures" is still very present in this area
- This practice is recognized by the majority as an activity requiring concentration and a certain intellectual effort.
- Very few people are specialists.

Kety knows a great number of procedures. With our first meeting, I had the feeling to be tested. The first string figure she proposed me to learn is, indeed, one of the most complicated procedures collected during my stay. It is about a figure in 3D named A*vestrus* representing an ostrich.

- Kety can, if you ask her, carry out the procedures very slowly by marking long breaks.
- Kety sees and represents in space the various operations or sub-procedures which are applied to the string.
- On several occasions, Kety was able to predict just by looking at the braid, without looking at what I had made whether or not I was going to be able to show the final figure.



Enrique (87 years old) was a Shaman and seems very erudite.

- He justified the absence of chants and songs connected to the procedures by saying to me that this activity is already interesting for itself. It is for him a difficult and intellectual activity.
- Enrique does not use inevitably the same fingers for repeating the same procedure again and again : he seems to concentrate more on the transformations of spatial configurations formed by the string, than on the fingers.

My methodology for collecting string figures

A symbolic language

to reduce a string figure to a simple formula

Example: ***Pu kava e hua***

[OpA][(di_1 → 2l)(pr_1 → 5l)(1 p+ 5n5f)(1rp)]
[((di_2 > 1l)(2 h+ pr.1f.2f.2n)(re1)][(1-2 gb di.2f)
(2 hr+)(ex)][(pr_1 > pr.2l)(1-2 gb di.2f)(2 hr+)(ex)]

Data bases / Data processing

Catalog for *Just Basic*

(String figures of Ua Pou)

1. **àu Kape + transformation**
2. **Uira**
3. **Kamo**
4. **Ua Huka**
5. **Nuku Hiva**
6. **Vai tahe mara**
7. **Série I (Koheao, Kohetua, O ana, èo)**
8. **Kivi**

M$(1)= « [Op A][re 1][(R1 h- 2l5n)(R2 h+ 5f)(re 1)][(pr_L2 > R2l)(L2 p+ 2n)(ex)][(pr.1→ 2l)(pr_1 > 5l)(1 p+ 5n)(1 rp)][(pr_1 > di.2l)(1 p+ di.2n)][NA 1][(re di.2l)(ex)][(pr_2 > 1l)(1l TF 2)])]**[(pr.1→ 2l)(pr_1 > 5l)(1 p+ 5n)(1 rp)][(pr_1 > di.2l)(1 p+ di.2n)][NA 1]** [(di_3 → di.2f)(di_3 > pr.2l)(3 p+ pr.2n)(re 5)(H vr+)]**[(re di.2l)(re 3)(ex)]**»

M$(2)= « [Op A][re 1][(pr_1→2l)(pr_1 > 5l)(1 p+ 5n)(re 5)][(pr_2 > 1l)(1l TF 2)][(di_1 > pr.2l)(1 p+ pr.2f)(di_1 > di.2l)(1 p+ di.2f)][(di_3 → di.2n)(pr_3 → pr.2n)(3 p+ pr.2n)][NA 1][H vr+] »

M$(3)= « [Op Ma1][(pr_L1 > 5l)(L1 p+ 5n)][(di_R1 > 2l)(R1 p+ 2f)][LH CA][(NA1)(re 5)][H hr-] »

M$(4)= «[(s on 1-2)(R2 h+ Rpa.s)(ex)][(pr_R1 p+ di.5f)(R1 rp)(repeat LH)][(2 hr+){(R2 h+ pr.R2f)}{(L2 h+ Lpr.1f)(re pr. L1l)}] [(pr_R1 p+ di.5f)(R1 rp)(repeat LH)][(2 hr+){(R2 h+ pr.R1f)}{(L2 h+ pr. L1f)}][(di_5 > 1l)(1l TF 5)(ex)]»

M$(5)= «[(s on L12R1)(R2 h+ Rpa.s)(ex)][(pr_1 p+ di.5f)(1 rp)][(2 hr+)(2 h+ pr.1f) (re pr.1l)}][(pr_1 p+ di.5f)(1 rp)][(2 hr+)(2 h+ pr.1f)][(di_5 > 1l)(1l TF 5)(ex)]»

M$(6)= « [Op A][re 5][(pr_2 > 1l)(1l TF 2)][(di_1 > pr.2l)(1 p+ pr.2f)(di_1 > di.2l)(1 p+ di.2f)][(di_3 → di.2n)(pr_3 → pr.2n)(3 p+ pr.2n)][NA 1][RH hr-] »

Marquesas

Elementary Operations	Freq	corrections + Op	%
p+	148	184	**20,58**
h+	13	13	1,45
h-	43	43	4,81
re	176	174	**19,46**
ex	96	126	**14,09**
rp	31	31	3,47
en	7	7	0,78
gb	68	80	**8,95**
gp	1	1	0,11
>	154	195	**21,81**
hr+	7	7	0,78
hr-	11	11	1,23
vr+	1	1	0,11
vr-	0	0	0,00
r+	1	9	1,01
r-	1	12	1,34
Total	758	894	100,00

Chaco

Elementary Operations	Freq	corrections + Op	%
p+	73	111	**25,40**
h+	19	21	4,81
h-	15	15	3,43
re	78	78	**17,85**
ex	39	61	**13,96**
rp	16	17	3,89
en	4	4	0,92
gb	11	13	2,97
gp	0	0	0,00
>	84	84	**19,22**
hr+	13	13	2,97
hr-	5	5	1,14
vr+	11	11	2,52
vr-	0	0	0,00
r+	2	2	0,46
r-	2	2	0,46
Total	372	437	100,00

arquesas

Sub-procedures	
Openings	
Op Ma1	1
Op Ma2	2
Op A	17
(Pos I')(Op A)	1

NA	11
TF	17
TF w	3
repeat LH	2
repeat RH	1
EX	1

spatialization		%
di_	50	39,68
pr_	76	60,32
	126	100,00

Chaco

Sub-procedures	
Openings	
Op B'	11
Op B"	1
Op A	16
Op A -1	1
0p A*	1

NA	7
TF	43
TF w	5
repeat LH	13
repeat RH	8
EX	6

spatialization		**Correction**
di_	113	142
pr_	109	

First outcomes

Fisrt data processings confirm that string games originating in certain traditional cultures, sometimes geographically and culturally distant, can use identical elementary operations.

However one should note, that the frequency of these operations may vary according to the region.

Observing the **order of succession** of elementary operations shows that a **sub-procedure comes from a choice of at least two elementary operations**: **this choice can be very different depending on the culture making string figures**.

Thus, the information collected reveals that the use of certain characteristic ”sub-procedures”, makes differences very clear within the overall corpus.

3. *Heart-sequence* : a topological view of a string figure

- Thomas Storer's mathematical work about string figures
- A functional language

Fingers = Fonctors

Sting and loops = Objects

Example : *Niu*

- The fingers are numbered from 1 (thumb) to up 5 (little finger).
- f = far ; n = near ; 1f is the far thumb string ; 5n the near little finger string.
- 1∞ : the loop of thumbs
- $\underline{O}$: Opening ; : Opening A
- $\vec{1}(\underline{2f})$ :: "1 go towards me up to 2n and pick up from bilow (proximally) far index string.
- # : return to position
- $\square 2$: release 2 loop.
- $\vec{1}\downarrow(2\infty)$: from above thumb insert into .
- $\succ 3\infty$ turn middle finger loop anticlockwise (for an observer located on the left side of the expert) ; $\prec 3\infty$: turn middle finger loop clockwise.

- $\longrightarrow$: to transfer
- $\succ 3\infty \rightarrow 5$: turn middle finger loop anti-clockwise and tranfer it to little finger

$$\underline{O}.A : \vec{1}(\underline{2f})\#\overleftarrow{3}(\underline{l1f})\#\square 1 \;\big|\vec{1}\downarrow(2\infty) : \underrightarrow{1}(\underline{5f})\#\square 5 \;\big|> \overrightarrow{3\infty} \rightarrow 5 : \square 2\# \;\big|$$

a. Heart-Sequence

- Tom Storer highlights that we can extract from this symbolic writing what he calls the *heart-sequence* of the procedure. This is based on geometry of loops.
- Then a string figure can be summarized in a certain number of passages of a loop into another.

Heart-sequence of *Niu*

$$\underline{O}.A:\left\{\begin{array}{l}\succ \overrightarrow{1\infty}\downarrow(2\infty):1\infty\rightarrow 5\\ \succ \underleftarrow{5\infty}\uparrow(2\infty):5\infty\rightarrow 1\end{array}\right\}:\square 2$$

b. Exploration of the Solomon string figure *Niu*

Comparison of the heart-sequences of string figures, which have a final figure very close to the one of *Niu*.

By very close, I mean that we can easily pass from one to an other by either a rotation of 180° in its plane (noted R), or a **reversal** (noted R) or, a mirror symmetry (noted S).

Example : *Pseudo-Niu*

By exchanging the role of thumb and little finger into the procedure *Niu*, we get a new procedure called *Pseudo-Niu* by T. Storer.

Heart-sequence of *Pseudo-Niu*

$$\underline{O}.A:\left\{\begin{array}{l}\prec \overleftarrow{5\infty}\downarrow(2\infty):5\infty\rightarrow 1\\ \prec \underrightarrow{1\infty}\uparrow(2\infty):1\infty\rightarrow 5\end{array}\right\}:\square 2$$

One can see that the final figure is almost the mirror image of the figure *Niu*. More precisely we have:

$$Pseudo\text{-}Niu = S\ o\ r\ (Niu)$$

This result can be generalized with all the string figures.

Equivalent final figures

Definition : in Knot Theory, two knots are equivalent if and only if they are isotope or if one is the mirror image *of the other*.

According to this definition, we will say that the final figures of both *Niu* and *Pseudo-Niu* are equivalent.

Pu kava (Marquesas Islands)

Heart-sequence of *Pu kava*

$$\underline{O}.A :: \succ \overrightarrow{1\infty}(5\infty) : \underleftarrow{1\infty}(5\infty) :\succ \overrightarrow{1\infty} \downarrow (2\infty) :: \square 2 : 1\infty \rightarrow 2$$

S o r (*Pu kava*) = *Niu*

Two procedures from which the heart-sequences seem different lead to the same final result.

Estrella (Chaco)

Estrella tends to confirm the assumption according to which, in certain cases (and perhaps always) it was indeed the operations on the loops as well as the braid thus generated which was seen and memorized by the practioners.

Heart sequence of *Estrella*

$$\underline{O}.A :: \prec \underline{1\infty}(5\infty) : \overleftarrow{1\infty}(5\infty) :\prec \underline{1\infty} \uparrow (2\infty) :: \square 2$$

Heart sequence of *Estrella*

$$\underline{O}.A :: \prec \underline{1\infty}(5\infty) : \overleftarrow{1\infty}(5\infty) :\prec \underline{1\infty} \uparrow (2\infty) :: \square 2$$

Indeed, the little difference between this last heart and *Pu kava* heart-sequence is that 1∞ rather begins its way by the lower part instead of the top. The “route” of 1∞ is the “symmetrical” (with respect to the plan of the figure) of the one revealed in the heart-sequence of *Pu kava*.

This generates a reversal of the final figure.

R o r (*Pu kava*) = *Estrella*

This formula is well explained by the comparison of the heart-sequences.

$$\underline{O}.A :: \succ \overrightarrow{1\infty}(5\infty) : \underleftarrow{1\infty}(5\infty) :\succ \overrightarrow{1\infty} \downarrow (2\infty) :: \square 2 : 1\infty \rightarrow 2$$

$$\underline{O}.A :: \prec \underrightarrow{1\infty}(5\infty) : \overleftarrow{1\infty}(5\infty) :\prec \underrightarrow{1\infty} \uparrow (2\infty) :: \square 2$$

Jasytata (Chaco)

The final figure of *Jasytata* is identical to the one of *Niu* (Solomon Islands).

However the heart-sequences are slightly different.

Heart-sequence of *Jasytata*

$$\underline{O}.A :\prec \underline{1\infty}(2\infty) : 1\infty \rightarrow 3$$

$$\succ \underline{5\infty} \uparrow (2\infty) : 5\infty \rightarrow 1 :\square 2$$

The second part of the formula above is also in the heart-sequence of *Niu*

$$\underline{O}.A : \left\{ \begin{array}{l} \succ \overrightarrow{1\infty} \downarrow (2\infty) : \underline{1\infty} \rightarrow 5 \\ \succ \underline{5\infty} \uparrow (2\infty) : \overleftarrow{5\infty} \rightarrow 1 \end{array} \right\} :\square 2$$

This suggests that the sequence $\underline{O}.A :\prec \underline{1\infty}(2\infty) : 1\infty \rightarrow 3$ of *Jasytata* leads to the same spatial configuration as the sub-procedure $\underline{O}.A :\succ \overrightarrow{1\infty} \downarrow (2\infty) : 1\infty \rightarrow 3$

Experiments show that it is indeed the case.

This highlights an interesting topological property of the configuration resulting from opening A.

Nepe (Solomon Islands)

The heart-sequence of Nepe is very close to the one of *Niu* and *Estrella.*

The first movements described by Maude (for Nepe) are the transfer of the thumb loops to the wrists. By disregarding hand, one can notice that the only transformation applied to configuration O.A is a reversal of the thumb loops.

In other words:

$$\succ \overrightarrow{2\infty} \downarrow (1\infty) :\succ \overrightarrow{5\infty} \downarrow (1\infty) :\succ 1\infty \rightarrow w$$

is equivalent to $\succ 1\infty$

Heart-sequence of *Nepe*

$$\underline{O}.A :\succ 1\infty : 1\infty \rightarrow w :: \underleftarrow{5\infty}(w\infty) : \overrightarrow{5\infty}(w\infty) :\succ \underrightarrow{5\infty} \uparrow (2\infty) ::\square 2$$

Comparison to the heart'sequence of *Estrella*

$$\underline{O}.A ::\prec \underrightarrow{1\infty}(5\infty) : \overleftarrow{1\infty}(5\infty) :\prec \underrightarrow{1\infty} \uparrow (2\infty) ::\square 2$$

By identifying w∞ with 1∞ in the heart of the procedure of *Nepe*, one sees clearly that the heart-sequence of *Estrella* is obtained from the *Nepe* by exchanging the roles of 1 and 5. It is thus natural to find the formula:

R o S (*Kapiva*) = *Estrella*

To conclude

I have found two types of heart-sequence for this figure "rhombus with double sides": the one of *Niu*, *Jasytata* and the one of *Nepe*, *Pu kava* and *Estrella*.

The final figures of *Niu* and *Nepe* are absolutely identical.

In a remarkable way, there are two different procedures, in the same place (Solomon islands) which lead to the same final figure even though they don't have the same "heart".

In the corpus I have collected in the Chaco, there is quite a similar phenomenon with *Estrella* and *Jasytata*. Except that the final figures are not identical but simply equivalent.

b. The concept of “prefix”

$$\underline{O}.A :\square 1 \mid \; \underrightarrow{1}\left(\overline{5f}\right)\#\vec{1}\left(\underline{2f}\right)\#\square 5 \; \mid \overleftarrow{5}\left(\underline{1f}\right)\#\square 1 \mid$$

This sequence leads, after opening it with a *Caroline extension,* to the final figure *2 rhombuses* .

The heart-sequence is given by :

$$\underline{O}.A :\square 1 ::\succ \underleftarrow{5\infty}\left(2\infty\right) :\succ \overrightarrow{5\infty}\downarrow\left(2\infty\right) ::$$

After performing these operations, if you put the braid on a plan and draw a bit the string, you may obtain the following drawing.

T. Storer calls it "*prefix*", in the sense that it will allow the opening of the final figure "2 *rhombuses*".

Other example

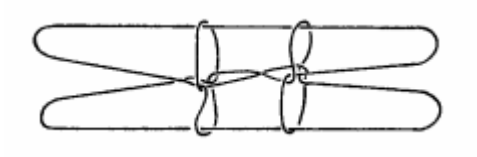
In performing the following heart,

$$\underline{O}.A :\square 1 :: \underleftarrow{5\infty}\left(2\infty\right) : \overrightarrow{5\infty}\downarrow\left(2\infty\right) ::$$

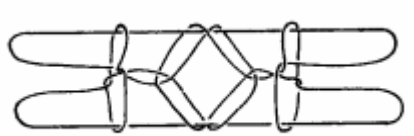
We get the prefix

c. The system of Solomon Islands patterns

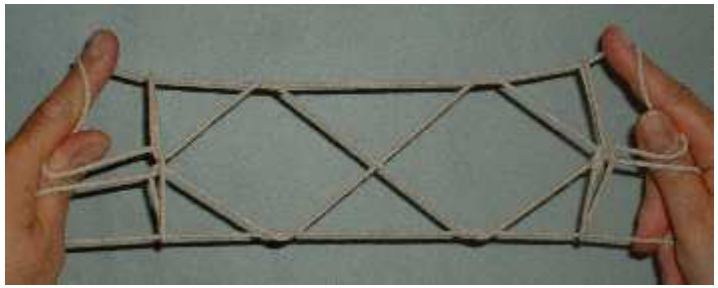
Roke nioke ke



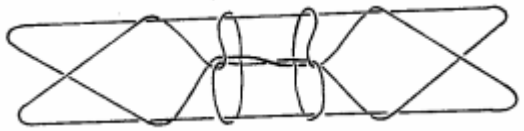
Namu



Repi susuhe'u

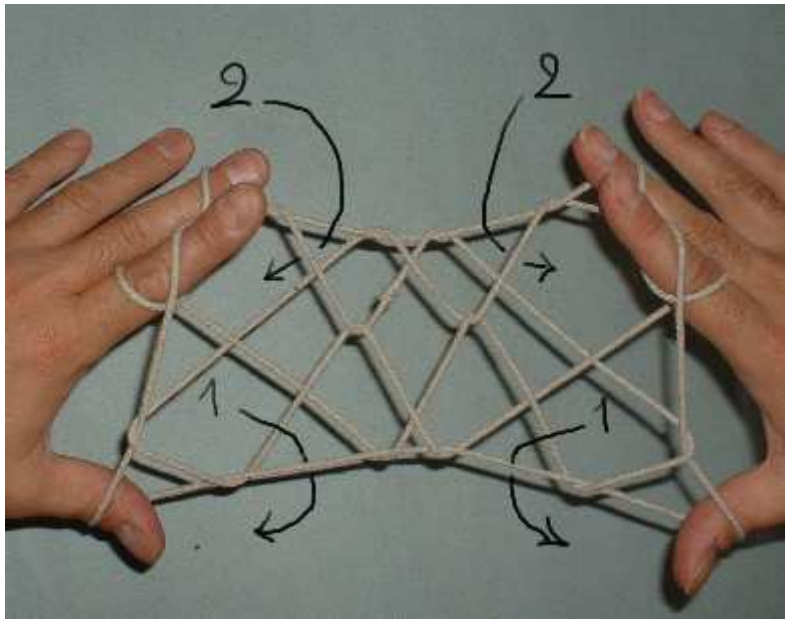


Whai wane

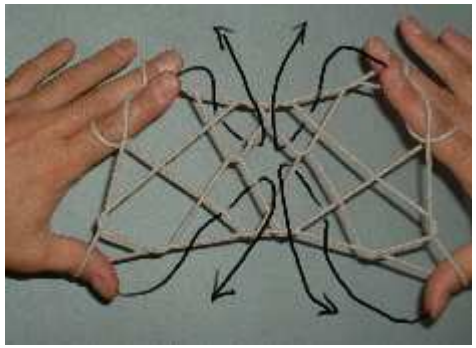


d. Understanding of the transformation *Au kape* to *Pu kava.*

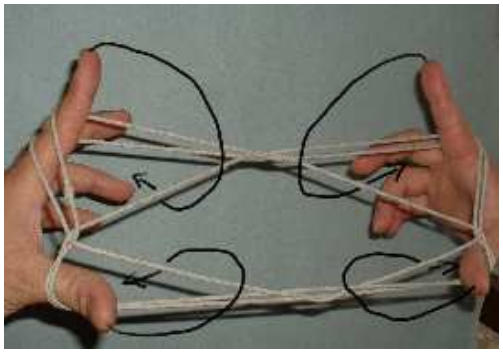
Au kape (Marquesas) is known throughout the Oceania



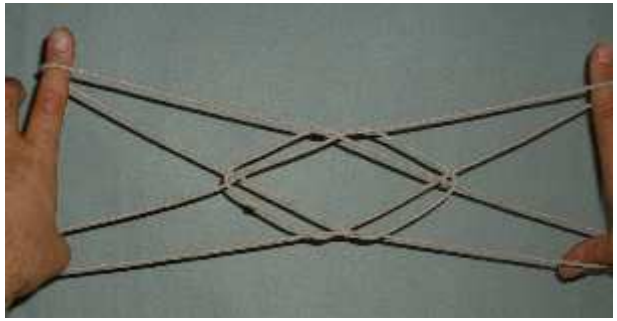
Transformation by “sewing”



1.



2.



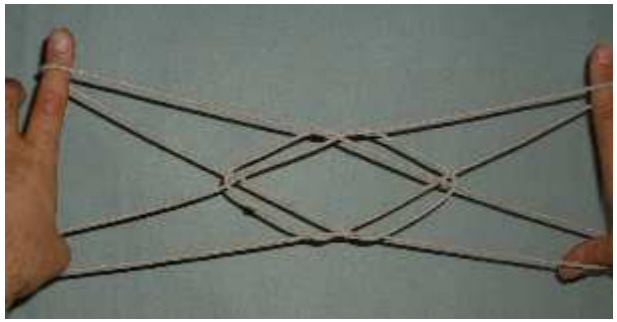
Heart- sequence of *Au kape* (beginning)

$$Config(A) : \underrightarrow{u2\infty}(l2\infty) : \underrightarrow{u2\infty} \uparrow (5\infty) : ....etc..$$

At this point if one applies the sequence :

$$u2\infty \rightarrow 1 :\square 5 :\succ 2\infty \rightarrow 5$$

A “lozenge with double sides” appears



Heart-sequence of Config(A)

$$Config(A) = \underline{O}.A : 2\infty \to 4 ::\prec \overleftarrow{5\infty} \downarrow (4\infty) : 5\infty \to 2 : 1\infty \to 2 :$$

Then, the heart sequence of the beginning of *Au kape:*

$$\underline{O}.A : 2\infty \to 4 ::\prec \overleftarrow{5\infty} \downarrow (4\infty) : 5\infty \to 2 : 4\infty \to 5 : 1\infty \to 2 : (config(A))$$

$$: \underline{u2\infty}(l2\infty) : \underline{u2\infty} \uparrow (5\infty) :$$

We can see that:

$$\prec \overleftarrow{5\infty} \downarrow (4\infty) \Leftrightarrow \prec \overleftarrow{5\infty} \downarrow (2\infty)$$

$$\underline{u2\infty} \uparrow (5\infty) \Leftrightarrow \underline{1\infty} \uparrow (2\infty)$$

So, the equivalence to *Niu*

$$\underline{O}.A : \left\{ \begin{array}{l} \prec \overleftarrow{5\infty} \downarrow (2\infty) : 5\infty \to 1 \\ \prec \underline{1\infty} \uparrow (2\infty) : 1\infty \to 5 \end{array} \right\} : \square 2$$

Transformation of *Au kape* into *Pu kava* by taking down few prefixes